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Corrigé type Maths 3 *duré 1:30 mn* *: Le 15 /01/2024*

Solution 1 *On a par partie*

$$I_1 = \int_0^1 x \ln \sqrt{x} dx = \frac{1}{2} \int_0^1 x \ln x dx$$

avec

$$\begin{cases} u = \ln x \rightarrow u' = \frac{1}{x} \\ v' = x \rightarrow v = \frac{1}{2}x^2 \end{cases} \quad \textbf{\textit{........0.5 pt}}$$

$$\begin{aligned} I_1 &= \frac{1}{2} \int_0^1 x \ln x dx = \lim_{t \to 0} \frac{1}{2} \int_t^1 x \ln x dx = \lim_{t \to 0} \left[\frac{1}{2} x^2 \ln x \right]_t^1 - \lim_{t \to 0} \frac{1}{4} \int_t^1 x dx \textbf{\textit{........0.5 pt}} \\ &= \lim_{t \to 0} \left[\frac{1}{2} x^2 \ln x - \frac{1}{8} x^2 \right]_t^1 = -\frac{1}{8} \textbf{\textit{........1.5 pt}} \end{aligned}$$

donc I_1 *est converge*
Pour le deuxieme integrale

$$I_2 = \int_0^{+\infty} \frac{1}{x\sqrt{x^2+1}} dx = \int_0^1 \frac{1}{x\sqrt{x^2+1}} dx + \int_1^{+\infty} \frac{1}{x\sqrt{x^2+1}} dx \textbf{\textit{........0.5 pt}}$$

au voisinage de 0 *on a*

$$x\sqrt{x^2+1} \approx x \textbf{\textit{........0.5 pt}}$$

et au voisinage ∞ *on a*

$$x\sqrt{x^2+1} \approx x^2 \textbf{\textit{........0.5 pt}}$$

on a d'apres l'integrale de riemman

$$\int_0^1 \frac{1}{x} dx \ \ (\alpha = 1) \ \ \textit{converge et} \ \int_1^{+\infty} \frac{1}{x^2} dx \quad (\alpha = 2) \ \ \textit{diverge} \textbf{\textit{........1.5 pt}}$$

donc

$$I_2 = \int_0^{+\infty} \frac{1}{x\sqrt{x^2+1}} dx \ \ \textit{est diverge}$$

Solution 2 *La solution homogène y_H*

$$\begin{aligned} y' - (\tan x)\, y &= 0 \Leftrightarrow \quad y' = (\tan x)\, y \\ &\Leftrightarrow \quad \frac{y'}{y} = \tan x \\ &\Leftrightarrow \quad \int \frac{y'}{y} dy = \int \frac{\sin x}{\cos x} dx \text{........}\textbf{\textit{0.5 pt}} \\ \ln |y| &= -\ln |\cos x| + C_1 \quad \textit{tel que } C_1 \in \mathbb{R} \\ \ln |y| &= \ln \frac{1}{|\cos x|} + C_1 \quad \textit{tel que } C_1 \in \mathbb{R} \end{aligned}$$

donc

$$y_H = \frac{C}{\cos x} \textit{el que } C \in \mathbb{R} \text{........}\textbf{\textit{0.5 pt}}$$

La solution particulière (la méthode de la variation de la constante)

$$y_p = \frac{C(x)}{\cos x} \text{........}\textbf{\textit{0.5 pt}}$$

donc

$$y_p' = \frac{C'(x)\cos x + C(x)\sin x}{\cos^2 x} \text{........}\textbf{\textit{0.5 pt}}$$

d'ou l'equation (E_1)

$$\begin{aligned} y' - (\tan x)\, y &= \sin x \Leftrightarrow \frac{C'(x)\cos x + C(x)\sin x}{\cos^2 x} - \frac{\sin x}{\cos x}\frac{C(x)}{\cos x} = \sin x \\ &\Leftrightarrow \quad \frac{C'(x)\cos x}{\cos^2 x} = \sin x \\ &\Leftrightarrow \quad C^{'}(x) = \sin x \cos x = \frac{1}{2}\sin 2x \text{........}\textbf{\textit{0.5 pt}} \end{aligned}$$

donc

$$\begin{aligned} C(x) &= \sin x \cos x = \frac{1}{2}\int \sin 2x dx \\ &= \frac{1}{2}\int \sin 2x dx = \frac{-1}{4}\cos 2x \text{........}\textbf{\textit{0.5 pt}} \end{aligned}$$

d'ou

$$y_p = y_H = \frac{C}{\cos x} + \frac{-1}{4}\cos 2x + C$$

par suite

$$y_G = y_H + y_p = \frac{-1}{4}\cos 2x + C \text{........}\textbf{\textit{0.5 pt}}$$

2- La somution homogène

$$y'' - 3y' + 2y = 0$$

E.C

$$r^2 - 3r + 2 = 0 \Rightarrow r_1 = 1 \, ou \; r_2 = 2 \boldsymbol{........0.5 \; pt}$$

donc

$$y_H = C_1 e^x + C_2 e^{2x} \quad tel \; que \; C_1, C_2 \in \mathbb{R} \boldsymbol{........0.5 \; pt}$$

La solution particulière est de la forme

$$y_p = ax^2 + bx + c \Rightarrow y' = 2ax + b \;\; et \; y'' = 2a \boldsymbol{........0.5 \; pt}$$

donc l'equation (E_2)

$$2a - 3(ax + b) + 2\left(ax^2 + bx + c\right) = 4x^2$$

donc

$$2ax^2 + (2b - 3a)x + 2c - 3b = 4x^2$$

par identification

$$\begin{cases} 2a = 4 \\ 2b - 3a = 0 \\ 2c - 3b = 0 \end{cases} \Rightarrow \begin{cases} a = 2 \\ b = 3 \\ c = \frac{9}{2} \end{cases} \boldsymbol{........0.5 \; pt}$$

$$y_p = C_1 e^x + C_2 e^{2x} + 2x^2 + 3x + \frac{9}{2} \quad tel \; que \; C_1, C_2 \in \mathbb{R} \boldsymbol{........0.5 \; pt}$$

donc

$$y_G = y_H + y_p = 2x^2 + 3x + \frac{9}{2}$$

Solution 3 *On a*

$$I_1 = \iint\limits_{D_1} (2x + 4y)\, dxdy \quad tel \; que \; D_1 = \left\{(x, y) \in \mathbb{R}^2 \; / 1 \leq x \leq 2 \;\; et \;\; 0 \leq y \leq 5\right\}$$

d'apres le theorem de Fubbini

$$\begin{aligned} I_1 &= \iint\limits_{D_1} (2x + 4y)\, dxdy = \int_0^5 \left(\int_1^2 (2x + 4y)\, dx \right) dy = \int_0^5 \left[x^2 + 4yx\right]_1^2 dy \boldsymbol{........0.5 \; pt} \\ &= \int_0^5 3 + 4y dy = \left[3y + 2y^2\right]_0^5 \boldsymbol{........0.5 \; pt} \\ &= 65 \boldsymbol{........1.5 \; pt} \end{aligned}$$

2- On a

$$I_2 = \iint\limits_{D_2} (x \cos y)\, dxdy \;\; tel \; que \; D_2 = \left\{(x, y) \in \mathbb{R}^2 \; / 0 \leq x \leq \sin y \;\; et \; 0 \leq y \leq \frac{\pi}{2}\right\}$$

d'apres le theorem de Fubbini

$$\begin{aligned}
I_2 &= \iint\limits_{D_2} (x\cos y)\,dxdy = \int_0^{\frac{\pi}{2}} \left(\int_0^{\sin y} (x\cos y)\,dx \right) dy \textit{\textbf{........0.5 pt}} \\
&= \int_0^{\frac{\pi}{2}} \left[\frac{x^2}{2}\cos y \right]_0^{\sin y} dy \\
&= \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin^2 y\cos y dy = \frac{1}{8}\int_0^{\frac{\pi}{2}} \sin^2 y\cos y dy \textit{\textbf{........0.5 pt}} \\
&= \frac{1}{8}\int_0^{\frac{\pi}{2}} \cos y - \cos 3y dy = \frac{1}{8}\left[\sin y - \frac{1}{3}\sin 3y \right]_0^{\frac{\pi}{2}} dy \textit{\textbf{........0.5 pt}} \\
&= \frac{1}{6} \textit{\textbf{........1 pt}}
\end{aligned}$$

Solution 4 *On a*

$$\begin{aligned}
\sum_{k=0}^{n} ka_k + (n+1)\,a_{n+1} &= \sum_{k=0}^{n+1} ka_k = \sum_{k=1}^{n+1} ka_k \textit{\textbf{........0.5 pt}} \\
&= \sum_{k=0}^{n} (k+1)\,a_{k+1} \textit{\textbf{........0.5 pt}} \\
&= \sum_{k=0}^{n} (k+1)\left(a_k + \frac{1}{k+1} \right) \textit{\textbf{........0.5 pt}} \\
&= \sum_{k=0}^{n} (k+1)\,a_k + 1 \textit{\textbf{........0.5 pt}} \\
&= \sum_{k=0}^{n} ka_k + \sum_{k=0}^{n} a_k + \sum_{k=0}^{n} 1 \textit{\textbf{........0.5 pt}}
\end{aligned}$$

donc

$$\sum_{k=0}^{n} a_k = (n+1)\,a_{n+1} - n \textit{\textbf{........2 pt}}$$